INVESTIGATION OF TURBULENCE IN CONNECTION WITH THE DETERMINATION OF THE ACOUSTIC CHARACTERISTICS OF A JET

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Results of experimental investigations of the characteristics of turbulence in a free isothermal jet are presented. The data obtained are used to calculate the acoustic characteristics of a jet: the acoustic power and its spectrum.

The basic equations describing the acoustic field produced by an isothermal turbulent jet of incompressible fluid were obtained by Lighthill [1, 2]. Using the equations of motion and continuity, Lighthill obtained a formula for the acoustic power per unit volume of the turbulent region:

$$\Delta W \sim V_i f^4 T_{ij}^2 / 4\pi a_0^5 \rho_0. \tag{1}$$

To use this formula, one must know all three components of the fluctuating flow velocity, the correlation between them, the volume of a typical turbulent eddy, and the frequency of the turbulent fluctuations. If these characteristics are known for all regions of the flow, one can determine the total acoustic power of the jet. It should be noted, however, that since, according to Lighthill, the main portion of the acoustic power is radiated from the initial section of the jet, detailed investigation of the characteristics of turbulence are required in that region. Unfortunately, the known papers dealing with investigations of this kind [3, 4] do not contain enough data to solve Lighthill's equations, and therefore additional investigations of the turbulent characteristics of a free jet are required. This paper gives the results of such investigations, in which attention has been paid mainly to the initial and transition sections.

The investigations were conducted with an air jet discharging from nozzles 40 and 55 mm in diameter with initial velocity u_0 = 25-135 m/sec and temperature T_0 = 288°K. Preliminary measurements with a Pitot tube indicated that the mean velocity profiles at various sections of these jets agreed sufficiently well with the universal profile [5].

The characteristics of turbulence were measured with a "Disa Electronic" equipment [6], consisting of two constant-temperature hot-wire anemometers and a correlator. Two types of probe were used — single and x-shape. The sensitive element of the two proves was a tungsten wire 0.005 mm in diameter and ~ 1 mm long. Spectral analysis of the turbulence was carried out with a spectrometer with a constant relative transmission band of 1/10 octave. The frequency characteristic was linear in the range 16-20 000 cps. The longitudinal and transverse correlation coefficients of the longitudinal component of velocity fluctuation were measured with two single-wire probes. One of the probes was fixed, while the other was traversed downstream along the jet axis (measuring the longitudinal correlation coefficient) or normal to the axis in the direction of the outer edge of the jet (measuring the transverse correlation coefficient).

The integral scale of turbulence was calculated from the formula

$$L = \int_0^{\xi_0} R\, d\xi, \quad R = \overline{u'(0)\, u'(\xi_0)} \Big/ \sqrt{\overline{u'^2}(0)}\, \sqrt{\overline{u'^2}(\xi_0)}\,. \tag{2}$$

Investigation of the fluctuation velocity field showed that in the initial and transition sections of the jet the distribution curves of intensity of turbulence $\sqrt{\overline{v'^2}}/u_0;\ \sqrt{\overline{u'^2}}/u_0;\ \sqrt{\overline{w'^2}}/u_0 = \varphi(y/r)$ have a maximum, the magnitude of which remains unchanged along a ray passing through the rim of the nozzle (i.e., y/r = 0). In the main section of the jet a reduction of the maximum value of intensity of turbulence is observed, and for relatively small values of x (when x/d < 20) the fall of the maximum value of $\sqrt{\overline{u'^2}}/u_0$ over the mixing region of the main section of the jet may be expressed by the relation $\sqrt{\overline{u'^2}}\,/u_0 = \mathrm{const}\, d/x$.

It is interesting to note that the law of variation of maximum intensity of turbulence over the mixing region of the jet corresponds to the law of variation of local mean velocity along a line passing through the rim of the nozzle. Thus, in the initial section of the jet the local mean velocity on the ray y/r = 0 remains fixed [5], while in the main section of the jet (for x/d < 20) it varies according to the law const/$\sqrt{x/d}$. This means that the wake behind the exit rim of the nozzle is the main factor in shaping the turbulence field in the region of the jet in question.

It should also be stressed that when the initial velocity of the jet was varied from 25 to 135 m/sec, the intensity of turbulence in the boundary layer of the jet barely changed.

The results of measuring all three fluctuation velocity components, expressed in dimensionless coordinates, enable one to describe each of them with sufficient accuracy by its generalized curve (Fig. 1). One such curve for the main section is presented in Fig. 2. It should be noted that whereas for the initial section the curves obtained are universal over the whole boundary layer, in the main section they are universal only in the region between y = 0 and the outer edge of the boundary layer; to get wholly universal curves in the main section of the jet, the axis would have to be taken as the origin of ordinates. However, since in solving Lighthill's equations it is more convenient to represent the characteristics of turbulence in all regions of the jet in a single coordinate system, coordinates were chosen in which the generalized curves are universal for the initial section (as the main contribution to the acoustic power comes from the initial section of the jet [2]).

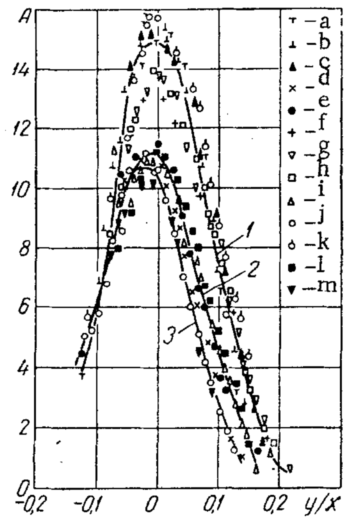

Fig. 1. Fluctuation velocity component fields in the initial and transition sections of the jet ($A\,(\%) \equiv \sqrt{\overline{u'^2}}/u_0 - 1;\ \sqrt{\overline{w'^2}}/u_0 - 2;\ \sqrt{\overline{v'^2}}/u_0 - 3$):

when d = 40 mm; u_0 = 120 m/sec (a – x/d = 2; b – 4; c – 5; d – 6; e – 4) and when d = = 55 mm, u_0 = 100 m/sec (f – x/d = 3; g – 4; h – 5; i – 6; j – 4; k – 2; l – 5; m – 5).

Correlation coefficients $R_{u'v'}$ for fluctuations of the longitudinal and radial velocity components, measured at various sections of the jet, are presented in Fig. 3. In spite of the considerable scatter, the variation of the correlation coefficient can also be described by a single universal curve. The variation of the coefficient of correlation between the longitudinal and tangential fluctuation velocity components shows that in the boundary layer of the initial section $R_{u'w'}$ is very small, and not one of the points in this region exceeds the value 0.1.

The results of the present investigations, and the data of others, support the conclusion that the integral scale of turbulence remains almost unchanged across the mixing region of the jet and increases linearly with axial distance x; the empirical relations derived in [3] and [4] $L_x = 0.13x$, $L_y = 0.036x$. then give quite good agreement with experiment.

Spectral analysis of the fluctuation velocity components showed that the nature of the turbelence spectra tends to change with increasing distance from the jet axis: the fraction of low-frequency components increases, while the high frequencies decrease. The form of the spectral curves varies only a little, however, and it may be assumed that the spectrum in a cross section of the mixing region of the jet remains unchanged. Sections further from the nozzle exit show a considerable change in the turbulence spectrum (Fig. 4). Variation of initial flow velocity also has a considerable effect. It may be seen from the graphs presented (Fig. 4) that at the same point in the flow the spectra of all three fluctuation velocity components are quite similar.

Analysis of the data obtained shows that the turbulence spectra can be represented in dimensionless form, if the Strouhal number Sh is used as dimensionless frequency. For the initial and transition sections of the jet $\mathrm{Sh} = f_x/u_0 =$ = 1.35, while for the main section $\mathrm{Sh} = fx\sqrt{x/x_n}/u_0 = 1.35$.

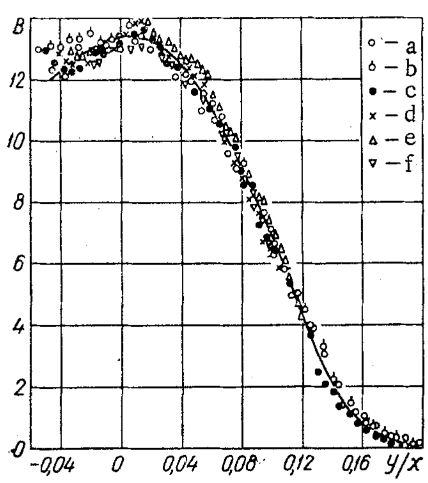

Fig. 2. Fields of longitudinal velocity component fluctuations in main section of jet ($B\,(\%) \equiv \sqrt{\overline{u'^2}}/u_0\sqrt{x/x_{\text{п}}}$) when d = 55 mm; u_0 = 100 m/sec; a – x/d = 6; b – 8; c – 10; d – 12; e – 15; f – 17.

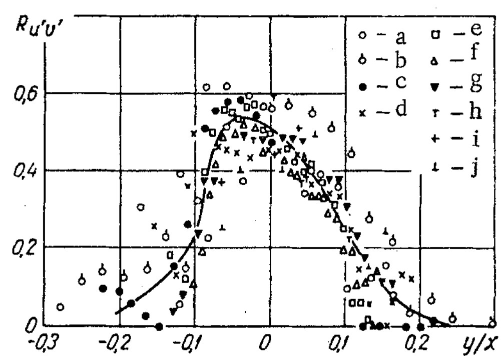

Fig. 3. Variation of correlation coefficient $R_{u'v'}$ in boundary layer of jet for d = 55 mm, u_0 = 100 m/sec (a – x/d = 1; b – 2; c – 3; d – 4; e – 5; f – 6) and for d = 40 mm, u_0 = = 120 m/sec; (g – x/d = 4; h – 3; i – 2; j – 1).

If we take as the characteristic dimension the boundary layer thickness at a given section, instead of the axial distance x, we obtain Sh ≈ 0.365.

The acoustic power radiated from an elementary section of the jet is

$$W_x = 2\pi\, dx \int_{y=-r}^{y_1} (r + y)\, dy\, \Delta W. \tag{3}$$

It is known that the initial section of the jet consists of a constant-velocity core and a boundary layer. Let us assume, as Lighthill does, that the constant-velocity core does not produce noise. This also follows from the data of the present investigation, which indicate that velocity fluctuations in that region are insignificant, and therefore the noise intensity given by (1) is also small. We may therefore take the ordinate of the inside edge of the jet y_2 as the lower limit of integration in (3).

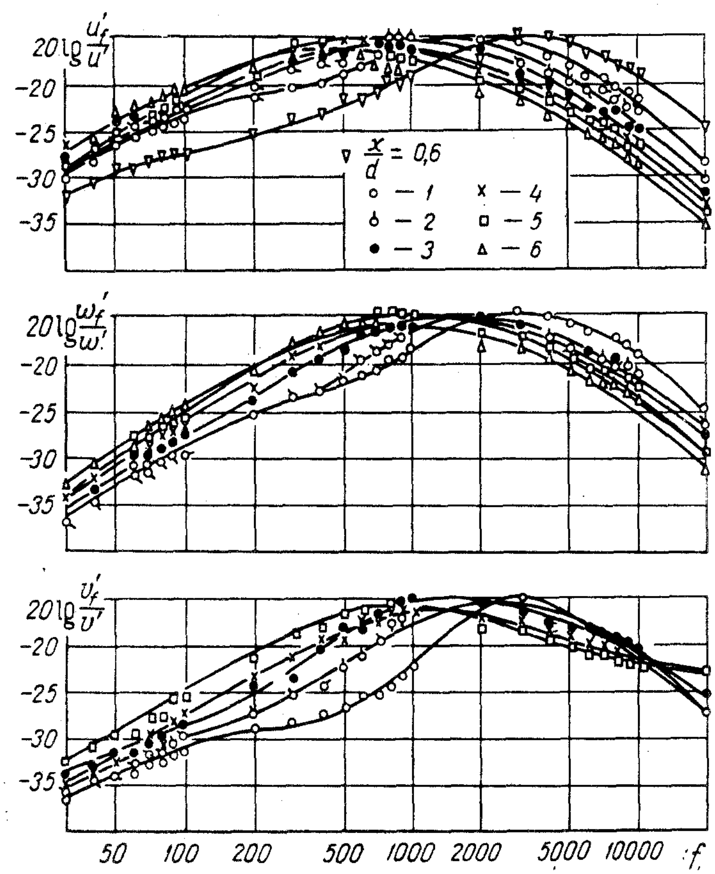

Fig. 4. Spectra of longitudinal, tangential and radial fluctuation velocity components in the initial and transition sections of the jet for d = 55 mm; u_0 = 100 m/sec.

Since the size of a typical eddy is determined by the region of positive correlation, we may write

$$V_l = L_x L_y L_z = k_1 x^3. \tag{4}$$

Using (1) and (4) and the value of the Sh number, we may write (3) for the initial section of the jet as

$$W_x = k\,\frac{u_0^4 dx}{\rho_0\, a_0^5 x} \int_{y_2}^{y_1} T_{ij}^2\, (r + y)\, dy. \tag{5}$$

Since all three components of velocity fluctuation and the corresponding correlation coefficients may be written in the form of dimensionless universal curves, the integral in (5) can easily be evaluated graphically. Then (5) becomes

$$W_x = k\,\frac{\rho_0\, u_0^8 dx}{a_0^5}\,(a_1 r + a_2 x). \tag{6}$$

Therefore, the noise power radiated by the initial section of the jet is given by

$$W_1 = k\frac{\rho_0 u_0^8}{a_0^5}\int\limits_0^{x_i}(a_1 r + a_2 x)\,dx = k\frac{\rho_0 u_0^8 d^2}{a_0^5}(2a_1 + 8a_2), \tag{7}$$

where $x_i = 4d$.

Analogously, we can evaluate the acoustic power radiated by an elementary section and by the whole region of the main and transition sections of the jet.

Thus, for the main section

$$W_x = k\frac{\rho_0 u_0^8 x_t^4 dx}{a_0 x^5}(a_3 r^2 + a_4 r x + a_5 x^2), \tag{8}$$

$$W_2 = k\frac{\rho_0 u_0^8 d^2}{a_0^5}\left(\frac{a_3}{16} + a_4 + 18a_5\right), \tag{9}$$

if we assume $x_t = 6d$.

For the transition section

$$W_x = k\frac{\rho_0 u_0^8 dx}{a_0^5}\left(\frac{a_3 r^2}{x} + a_4 r + a_5 x\right), \tag{10}$$

$$W_3 = k\frac{\rho_0 u_0^8 d^2}{a_0^5}(0.101a_3 + a_4 + 10a_5). \tag{11}$$

The values of the coefficients in (6)-(11), determined by graphical integration, are:

$$a_1 = 0.148\cdot10^{-3};\quad a_2 = 0.525\cdot10^{-5};\quad a_3 = 0.398\cdot10^{-3};$$
$$a_4 = 0.561\cdot10^{-4};\quad a_5 = 0.192\cdot10^{-5}.$$

The acoustic power of the whole jet is

$$W_0 = W_1 + W_2 + W_3. \tag{12}$$

Calculation according to (12) indicates that the acoustic power of the initial and transition sections is approximately 75% of the total power. Thus, most of the noise comes from sections of the jet less than six diameters from the end of the nozzle.

Using (6), (8), and (10), we can calculate the acoustic power spectrum of the jet. Since the acoustic power is proportional to the fourth power of the fluctuation velocity, the spectrum of the acoustic power radiated by a given section of the jet will have a maximum that is much more sharply expressed than the maximum in the turbulence spectrum. It may therefore be assumed that the acoustic power spectrum of an elementary section is contained within quite a narrow band, whose mean frequency corresponds to the frequency of the maximum component of the turbulence spectrum. The dependence of this mean frequency on the distance x is expressed by values of the Sh number.

The acoustic power spectrum for a jet of diameter 100 mm calculated in this way showed good agreement with the experimental data of [7].

NOTATION

W – acoustic power; ρ_0 – atmospheric air density; V_l – volume of a typical eddy; f – fluctuation frequency; T_{ij} – turbulent friction stress tensor; d = 2r – nozzle diameter; u_0 – initial flow velocity; T_0 – total temperature of flow; u', v', w' – respectively, longitudinal, radial, and tangential components of fluctuation velocity; L – integral scale of turbulence; R – coefficient of correlation between longitudinal components of fluctuation flow velocity u', measured at two points a distance ξ apart; ξ_0 – distance between the wires of anemometer probes for which R vanishes; y – distance in jet cross section from rim of nozzle to point investigated, the positive direction being toward the outer edge of the jet; x – axial distance from lip of nozzle to point investigated; $R_{u'v'}$, $R_{u'w'}$ – single-point correlation coefficients between two corresponding components of the fluctuation velocity; L_x, L_y – respectively, longitudinal and transverse integral scales of turbulence; x_i, x_t – abscissas of the ends of the initial and transition sections of the jet; y_1, y_2 – ordinates of the outside and inside edges of the jet.

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